

Intensity Duration Frequency Relations

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Roscoff, 30.030.2025





Urban Drainage Design determine capacity of
drainage system or components



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Flood Risk Assessment due to rainfall, knowing exceedance probabilities

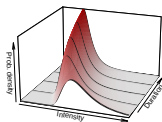


Urban Drainage Design determine capacity of drainage system or components

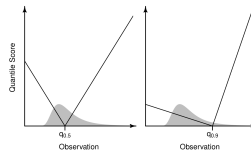
Flood Risk Assessment due to rainfall, knowing exceedance probabilities

Agricultural Planning design drainage/irrigation systems, protecting crops

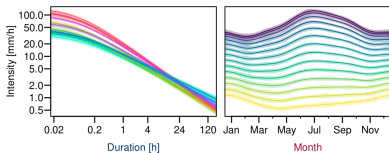




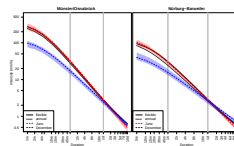
Duration-dep. GEV



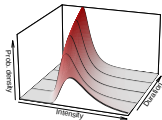
Quantile Score



Seasonal IDF



Flexible IDF

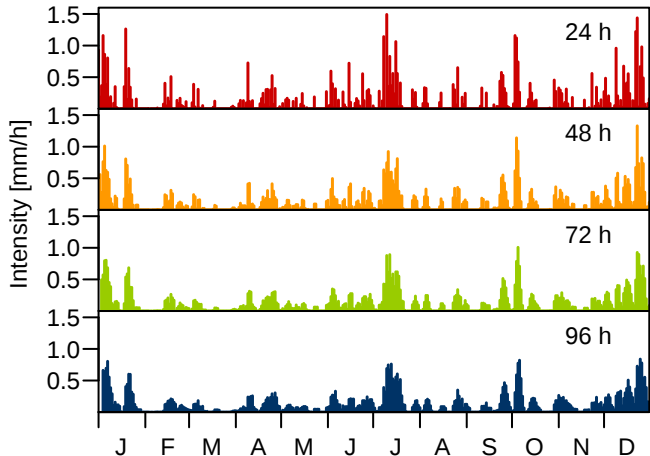


Duration-dependent GEV

Extreme values for different durations

- Daily data
- Calculate precipitation intensities I for different durations d

$$I(t, d) = \frac{1}{d} \sum_{t'=t-d}^t pr(t')$$

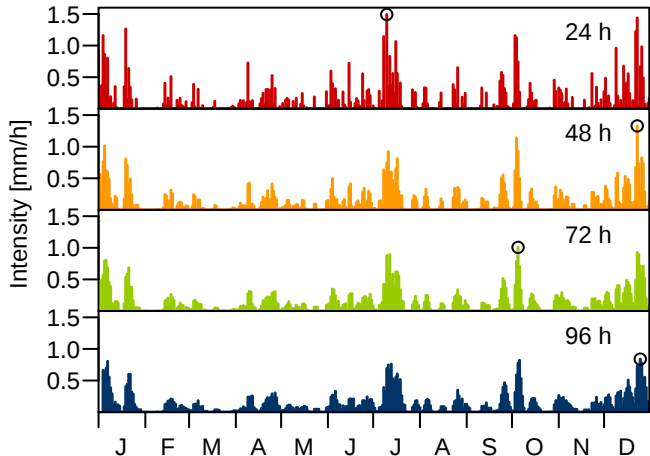


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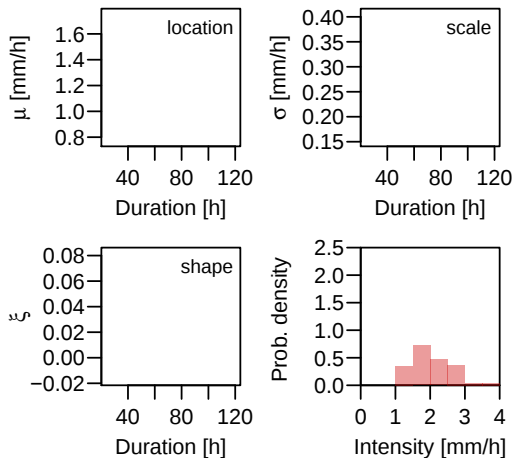
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- ▶ Extreme values:
 - ▶ Block maxima approach
- e.g. Annual maxima for different durations



Estimate GEV for different durations

Duration [h] — 24

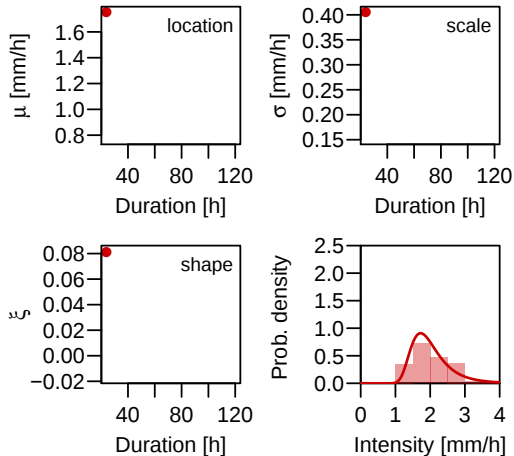


$$G(I) = \exp \left\{ - \left[1 + \xi \left(\frac{I - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}$$

- ▶ GEV: modeling distribution of annual intensity maxima
- ▶ Separately for each duration

Estimate GEV for different durations

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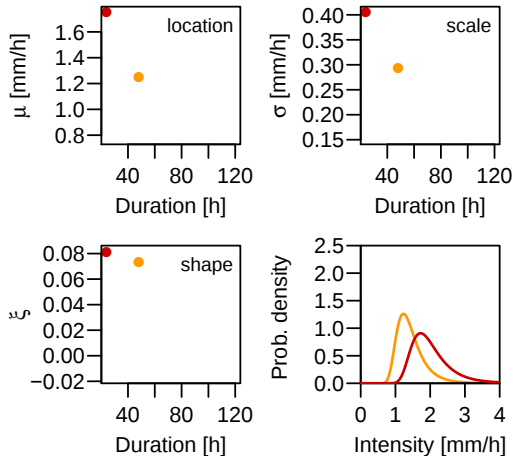


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Estimate GEV for different durations

Duration [h] — 24 — 48

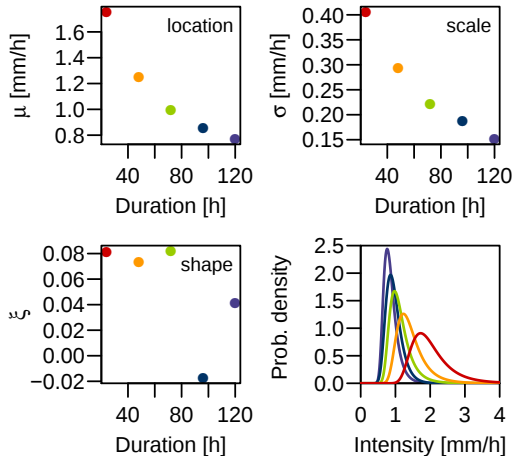


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Estimate GEV for different durations

Duration [h] — 24 — 48 — 72 — 96 — 120



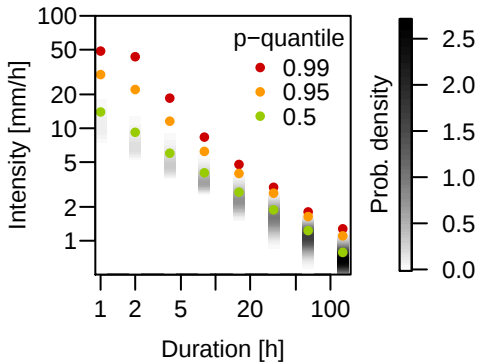
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- ▶ Separately for each duration

IDF curves

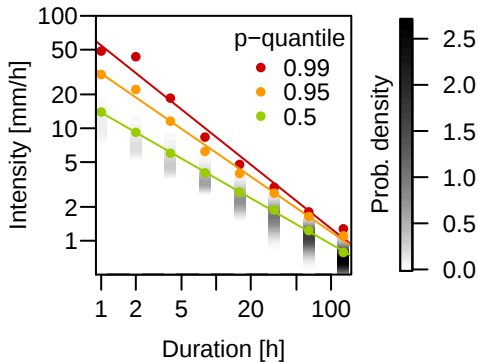
Typical approach (Engineers, DWD)

- ▶ 2 separate statistical models
 1. EVD (GEV) for different durations



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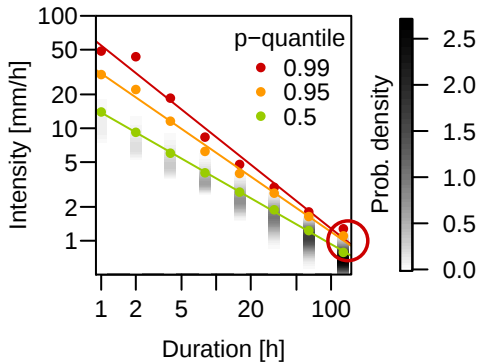
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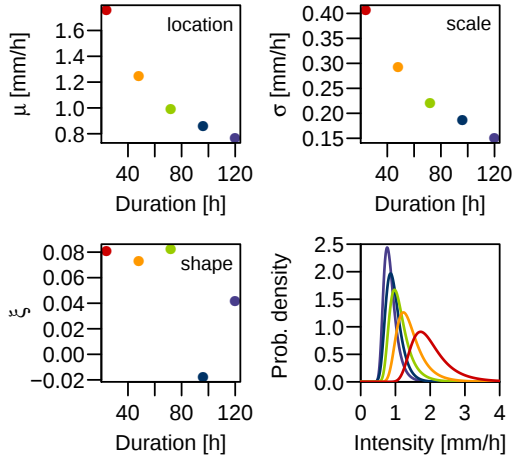
Typical approach (Engineers, DWD)

- ▶ 2 separate statistical models
 1. EVD (GEV) for different durations
 2. quantiles smooth function of duration
- ▶ crossing of quantiles
- **Inconsistent**



Duration-dependent GEV

Duration [h] — 24 — 48 — 72 — 96 — 120

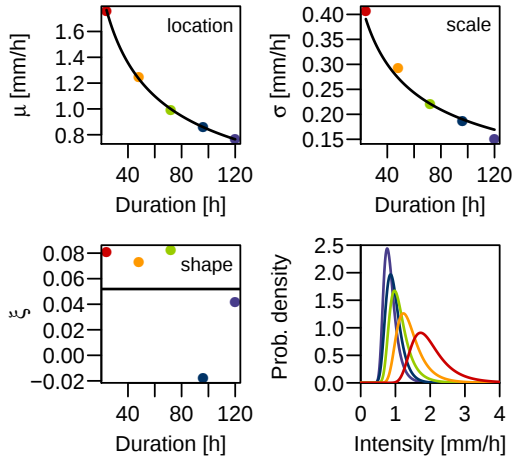


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- Idea: model dependence of GEV parameters on duration

Duration-dependent GEV

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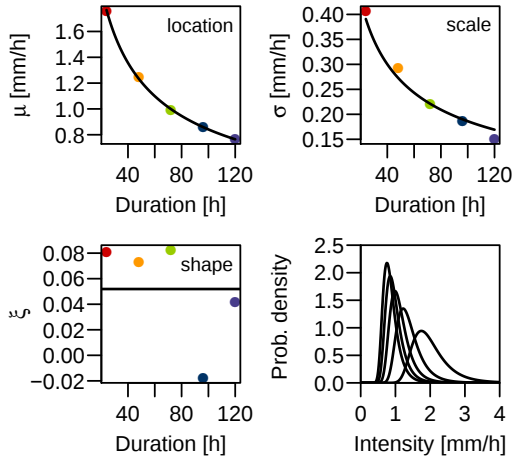


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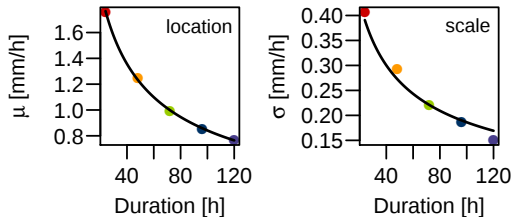
- Idea: model dependence of GEV parameters on duration
- **Model all durations at once**

Duration-dependent GEV

- ▶ Assumptions for d -dependence of μ , σ , ξ by Koutsoyiannis et al., 1998, J. Hydrol., 206:

Duration-dependent GEV

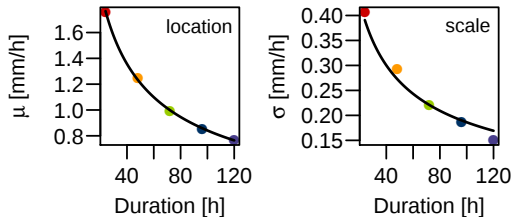
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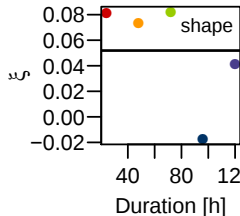
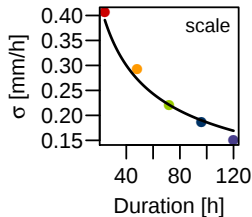
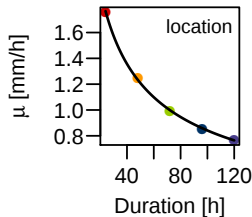
- location and scale show similar behavior
- use 2 additional parameters: θ and η

$$\mu(d) = \tilde{\mu} \sigma(d)$$

$$\sigma(d) = \sigma_0 (d + \theta)^{-\eta}$$

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$$\xi \neq f(d)$$

Duration-Dependent GEV (d-GEV)

$$G(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}$$

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→ assumptions for parameters ¹

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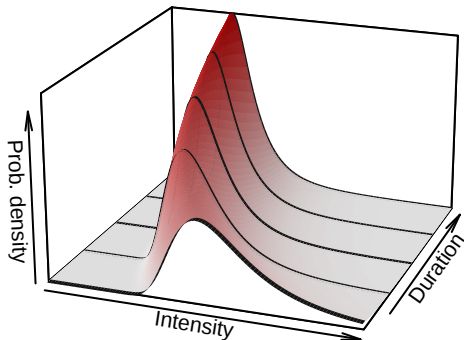
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→ one d-GEV for all durations:

$$G(z, d) = \exp \left\{ - \left[1 + \xi \left(\frac{z}{\sigma_0 \cdot (d + \theta)^{-\eta}} - \tilde{\mu} \right) \right]^{-1/\xi} \right\}$$



¹Koutsoyiannis et al. *J. Hydrol.* 1998, 206:118–135.

Consistent and parsimonious IDF curves

What we expect (pros)

- ▶ more efficient use of the data
 - reduced uncertainty
 - more complexity possible for modelling
 - ▶ seasonal cycle
 - ▶ spatial variability
 - ▶ climate change
- ▶ consistency (no quantile crossing)

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What we expect (cons)

- ▶ dependence between durations → problems
 - Jurado et al., *Water* 2020, 12(12), 3314; <https://doi.org/10.3390/w12123314>

Summary and Further Questions

Summary

- ▶ Model intensity maxima for all durations simultaneously → **d-GEV**
- one model with consistent quantiles (instead of 2)
- ▶ d-GEV (+ generalized linear modelling of parameters) implemented in **R-Package 'IDF'**

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→ **compare to individual durations (QVS)**
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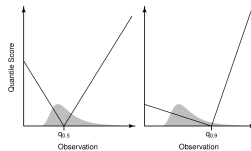
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→ **seasonal and spatial covariates**



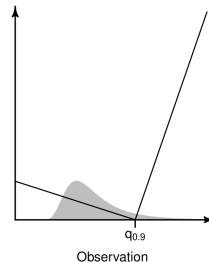
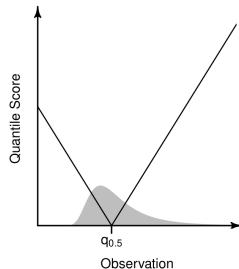
Quantile verification score

For exceedance probability p

$$QS(p) = \frac{1}{N} \sum_{n=1}^N \rho_p(o_n - q_p),$$

$$\rho_p(u) = \begin{cases} pu & , u \geq 0 \\ (p-1)u & , u < 0. \end{cases}$$

$QS \geq 0$, negatively oriented, $QS(p)_{\text{pef}} = 0$



Quantile Skill Score

$$\text{Skill Score} = \frac{QS(p) - QS(p)_{\text{ref}}}{QS(p)_{\text{perf}} - QS(p)_{\text{ref}}}$$

with $QS(p)_{\text{perf}} = 0$

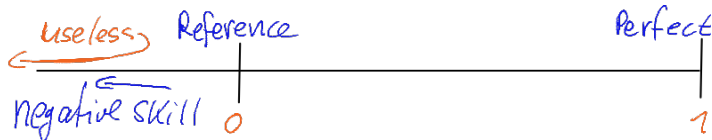
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Model improvement over reference

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Quantile Skill Index

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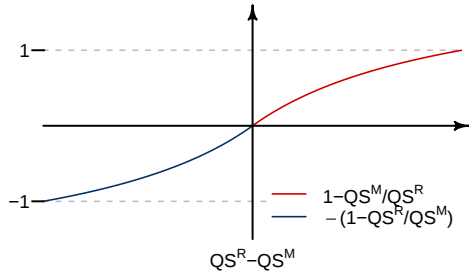
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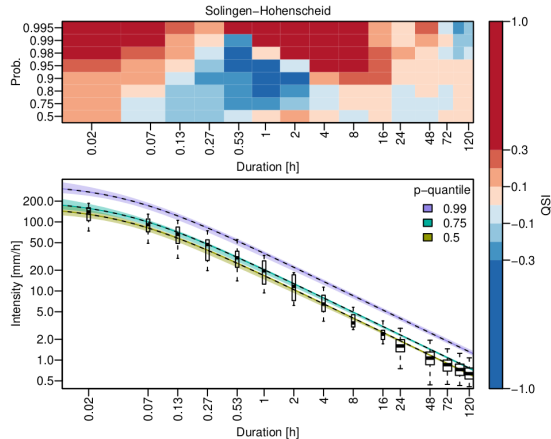
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"Quantile Skill Index combine both informations:

$$QSI_{s,d}(p) = \begin{cases} QSS_{s,d}^M(p) & , QS_{s,d}^M(p) \leq QS_{s,d}^R(p) \\ QSS_{s,d}^R(p) & , QS_{s,d}^M(p) > QS_{s,d}^R(p) \end{cases}$$





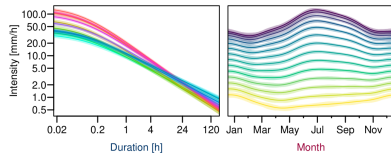
Solingen-Hohenscheid, Wupperverband, d-GEV with spatial covariates.
QSI vs. individual durations GEV for single station

Summary

- ▶ Quantiles are the goal, detailed verification for
 - ▶ all probabilities / return periods
 - ▶ durations
- ▶ use in cross validation setting

Reference

Estimating IDF curves consistently over durations with spatial covariates.
J. Ulrich, O. Jurado, M. Peter, M. Scheibel and H. Rust
Water, 12(11), 3119
<https://www.mdpi.com/2073-4441/12/11/3119>



Seasonal IDF

Monthly Maxima and Seasonally Varying Parameters

- Model **monthly maxima** for range of **durations** using d-GEV

$$G(z, d; \tilde{\mu}, \sigma_0, \xi, \theta, \eta)$$

Monthly Maxima and Seasonally Varying Parameters

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- Sum of trigonometric functions for all parameters $\phi \in \{\tilde{\mu}, \sigma_0, \xi, \theta, \eta\}$:

$$\phi(m) = \phi_0 + \sum_{j=1}^J \left[\beta_j^\phi \sin(j\omega m) + \gamma_j^\phi \cos(j\omega m) \right], \quad \begin{array}{l} \text{with } \omega = 2\pi/12 \\ \text{and } m: \text{ month of the year} \end{array}$$

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Monthly Maxima and Seasonally Varying Parameters

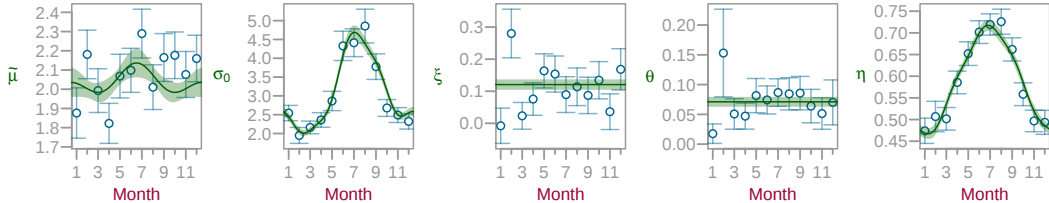
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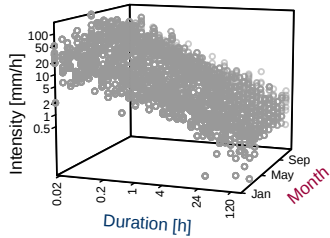
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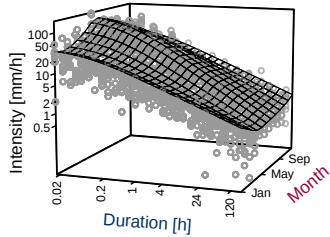
○ separate d-GEV for every month
— d-GEV with monthly covariates

How does the IDF relationship vary throughout the year?



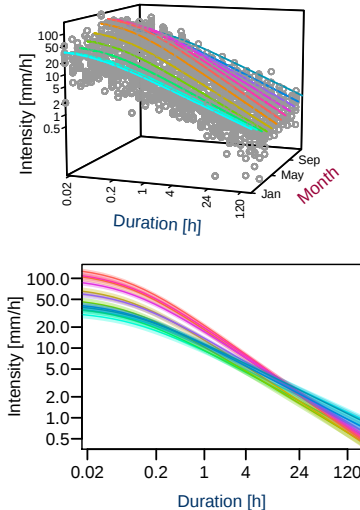
- Example station
Bever-Talsperre

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- ▶ Example station
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- ▶ Estimated 0.9-quantile

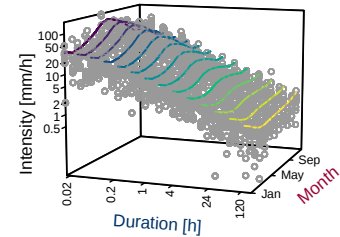
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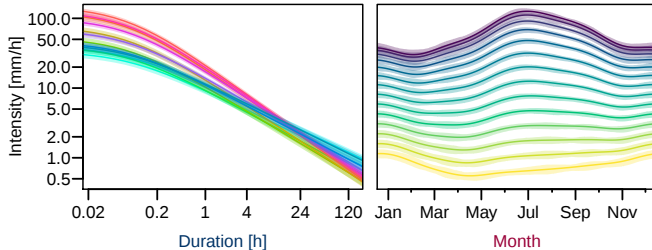
- ▶ Example station
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- ▶ For each **month**

- ▶ IDF-curves
 - ▶ more steep
in summer months

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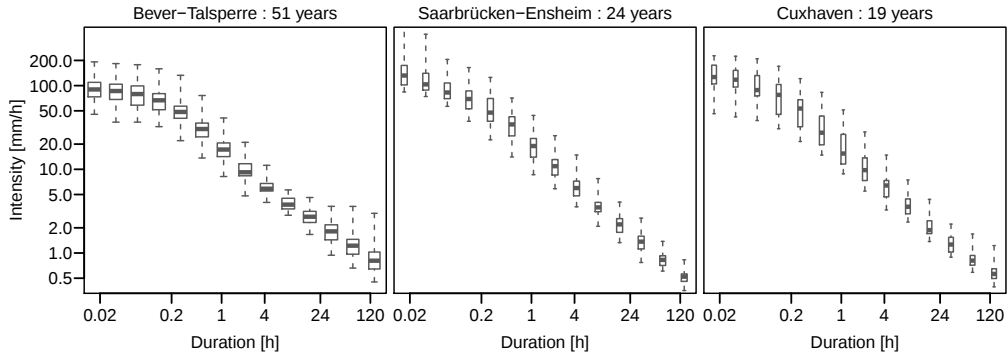
- ▶ Example station
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- ▶ For each **month**
- ▶ For each **aggregation level**



- ▶ IDF-curves
 - ▶ more steep
in summer months
- ▶ Intensity maxima
 - ▶ short durations:
summer
 - ▶ long durations:
autumn/ winter

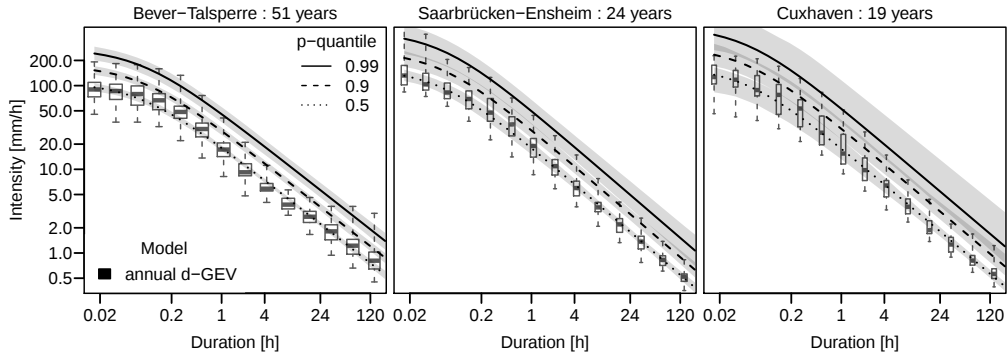
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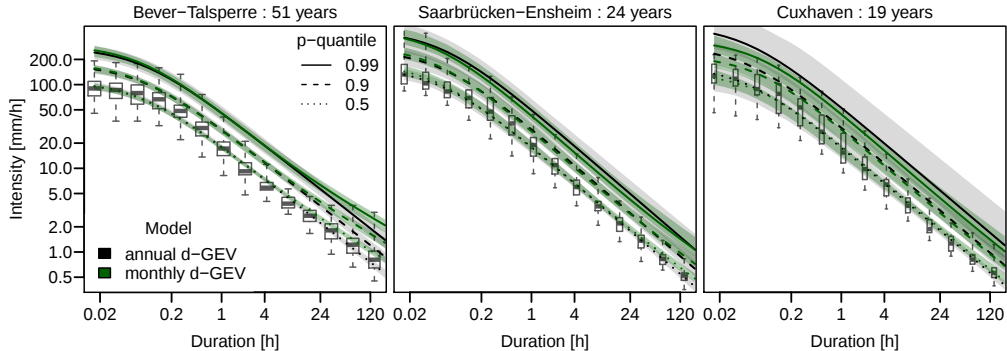
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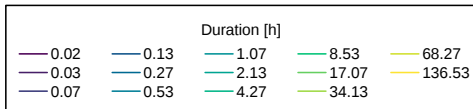
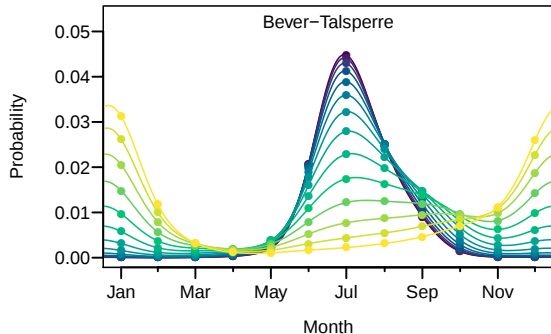
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i.e. different length of time series:
- ▶ Estimate annual IDF curves based on
 - ▶ Annual maxima
 - ▶ Monthly maxima

- ▶ Reduced uncertainties
- ▶ Less restricted dependence on duration



When in the year do annual maxima occur?

- Probability that annual 0.9-quantile is exceeded in certain month

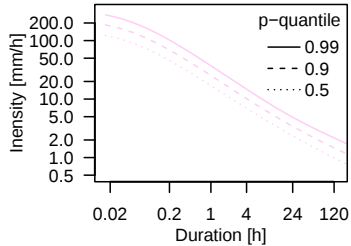


- Maximum probability shifts
 - from summer for short durations (convective events)
 - to winter for long durations (frontal events)
- Seasonality of long-lasting events depends on location

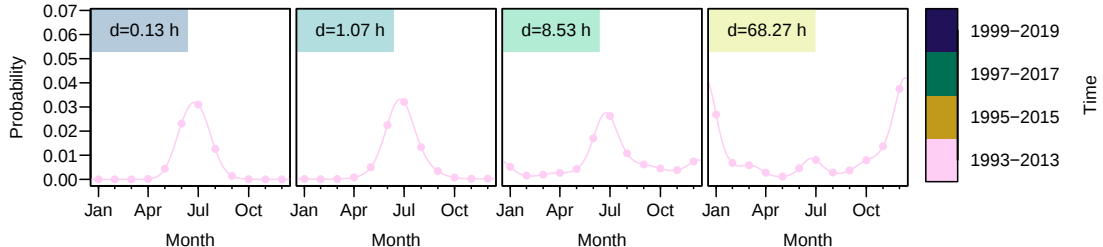
Outlook

How do Intensity and seasonality change over time?

Schmücke



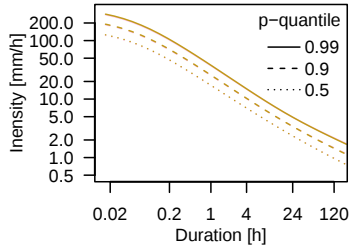
Moving 20 year time window



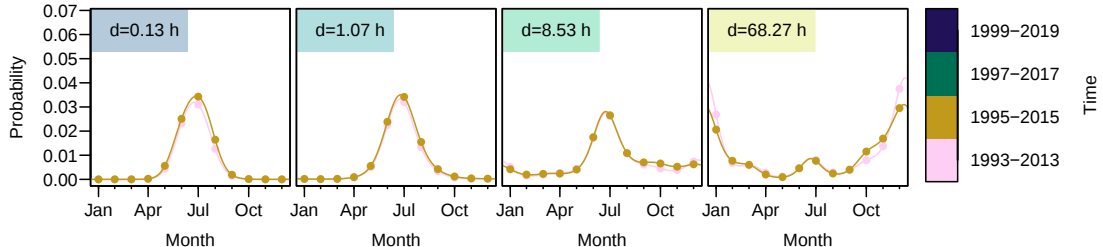
Outlook

How do Intensity and seasonality change over time?

Schmücke



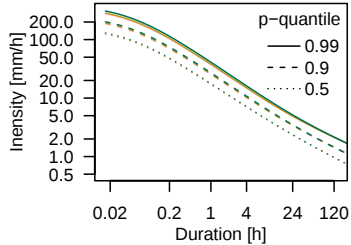
Moving 20 year time window



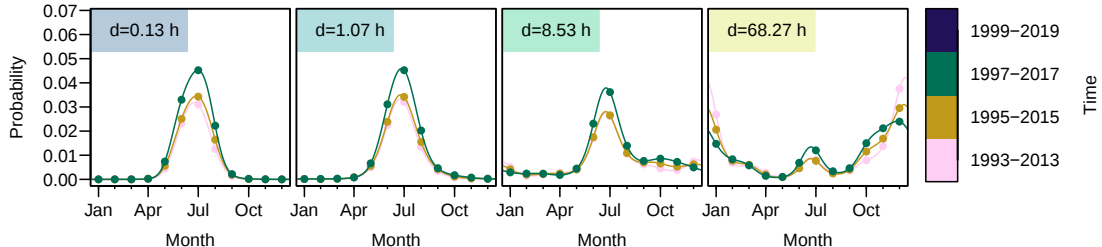
Outlook

How do Intensity and seasonality change over time?

Schmücke



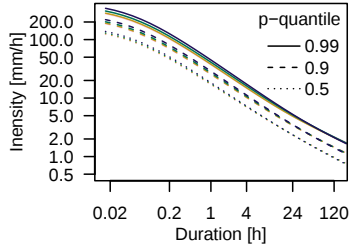
Moving 20 year time window



Outlook

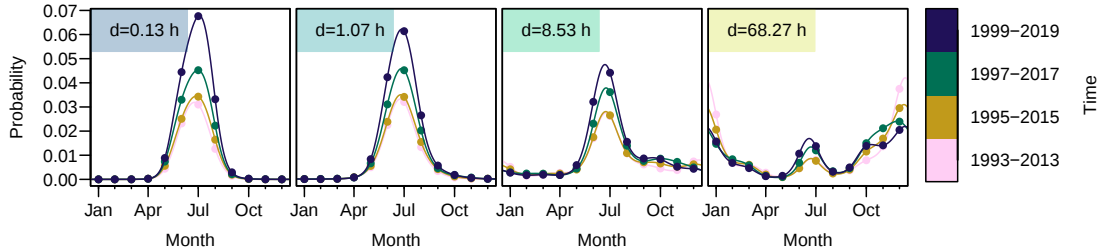
How does Intensity and seasonality change over time?

Schmücke



Moving 20 year time window

- ▶ short durations: increasing intensities
- ▶ long durations: changes in seasonality



Summary and Reference

Summary

- ▶ reduced uncertainty (← esp. short series)
- ▶ increase flexibility (← esp. long series)

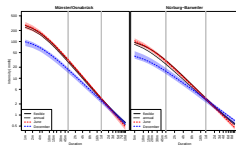
Reference

Modeling seasonal variations of extreme rainfall
on different time scales in Germany

J. Ulrich, F. Fauer, H. Rust

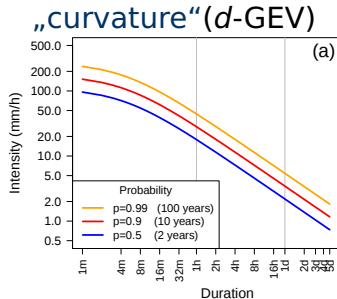
Hydrology and Earth System Sciences 25.12 (2021): 6133-6149

<https://hess.copernicus.org/articles/25/6133/2021/>



Flexible IDF

Add flexibility to d-GEV



$$G(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu(d)}{\sigma(d)} \right) \right]^{-1/\xi} \right\}$$

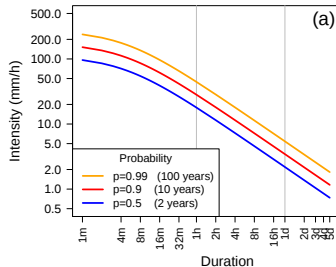
$$\mu(d) = \tilde{\mu} (\sigma_0 (d + \theta)^{-\eta})$$

$$\sigma(d) = \sigma_0 (d + \theta)^{-\eta}$$

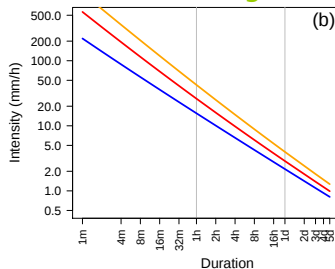
$$\xi = \xi_0$$

Add flexibility to d-GEV

„curvature“ (d-GEV)



„multiscaling“



$$G(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu(d)}{\sigma(d)} \right) \right]^{-1/\xi} \right\}$$

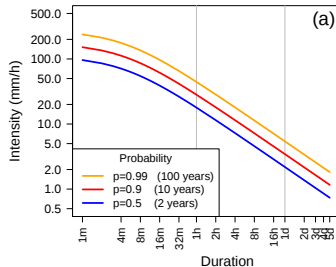
$$\mu(d) = \tilde{\mu} (\sigma_0 (d + \theta)^{-\eta})$$

$$\sigma(d) = \sigma_0 (d + \theta)^{-\eta - \eta_2}$$

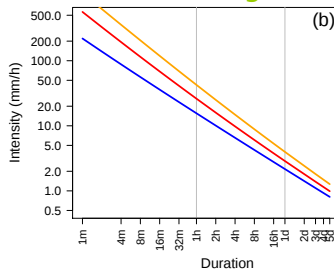
$$\xi = \xi_0$$

Add flexibility to d-GEV

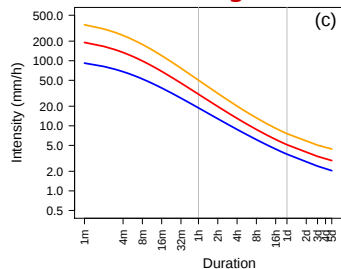
„curvature“ (d-GEV)



„multiscaling“



„flattening“



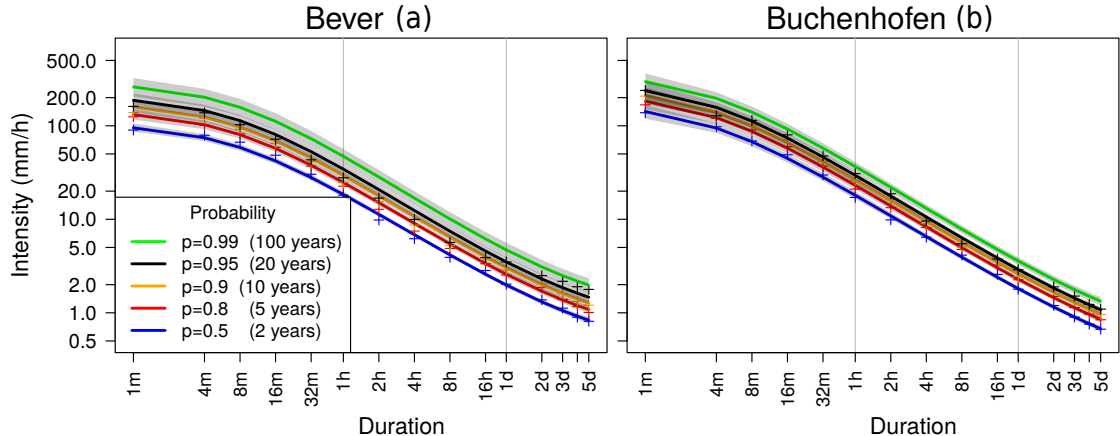
$$G(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu(d)}{\sigma(d)} \right) \right]^{-1/\xi} \right\}$$

$$\mu(d) = \tilde{\mu} (\sigma_0 (d + \theta)^{-\eta} + \tau)$$

$$\sigma(d) = \sigma_0 (d + \theta)^{-\eta - \eta_2} + \tau$$

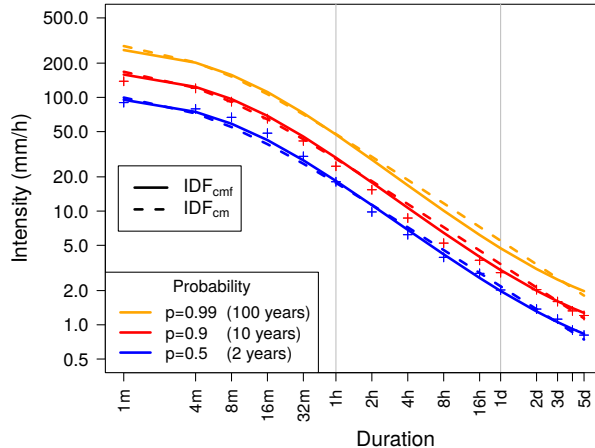
$$\xi = \xi_0$$

Flexible IDF example: Bever and Buchenhofen

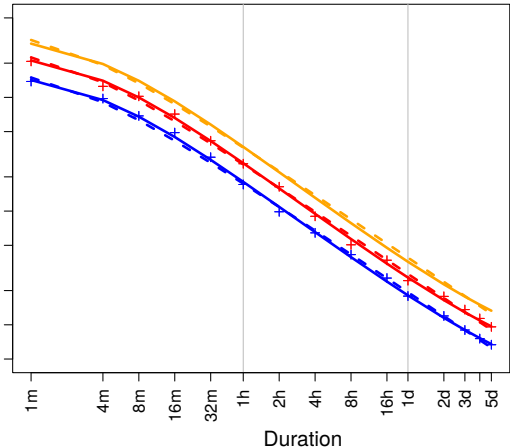


Flexible IDF example: Bever and Buchenhofen

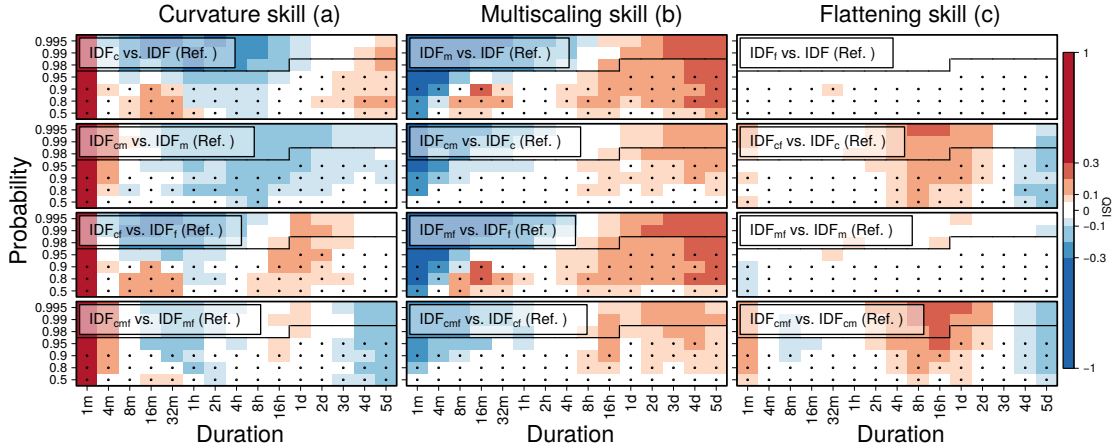
Bever (a)



Buchenhofen (b)

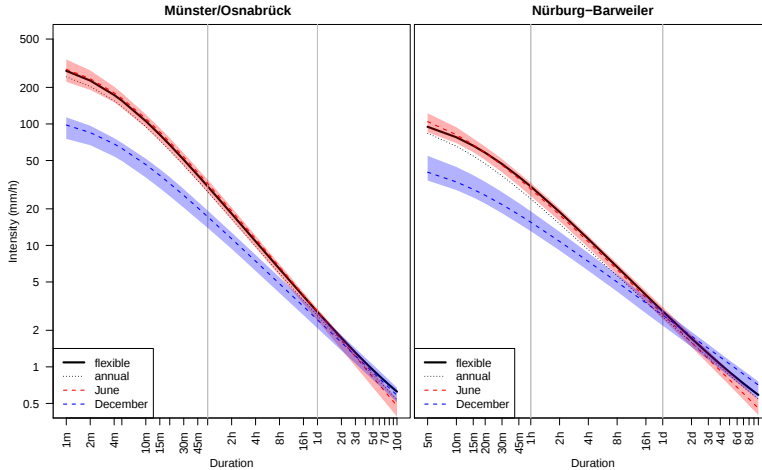


Improvement due to individual features



Average over 92 station in the Wupper catchment.

Flexible d-GEV accounts for various Processes



Summary

- ▶ added flexibility can be useful
- ▶ „flattening“ needed when mixing processes with different slopes η (seasons)

Reference

Flexible and consistent quantile estimation for intensity–duration–frequency curves
F. Fauer, J. Ulrich, H. Rust
Hydrology and Earth System Sciences, 25.12 (2021): 6479-6494
<https://hess.copernicus.org/articles/25/6479/2021/>

Summary and Outlook

Summary

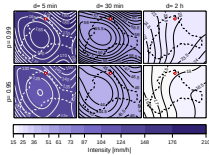
- ▶ d-GEV to exploit smoothness across durations
- ▶ detailed verification with QVS
- ▶ seasonal IDF curves
- ▶ added flexibility (account for mixture of processes)

Outlook

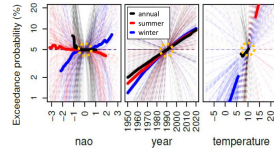
- ▶ projecting IDF relations based on global climate models
- ▶ extended GEV to compensate for deviations from GEV
- ▶ combine grid-based and gauge data
- ▶ use smooth (GAM-like) relations instead of parametric

IDF

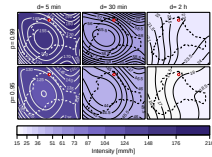
<https://cran.r-project.org/web/packages/IDF/index.html>



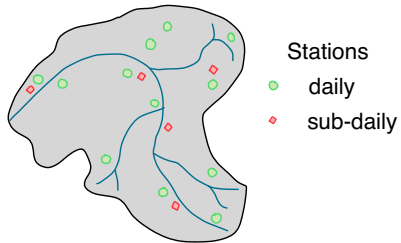
Spatial covariates

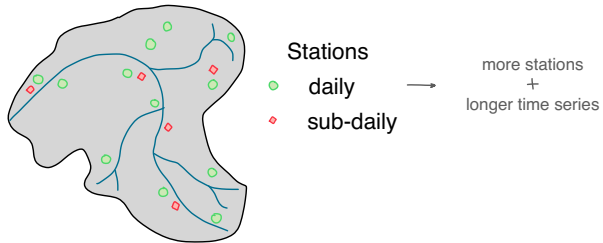


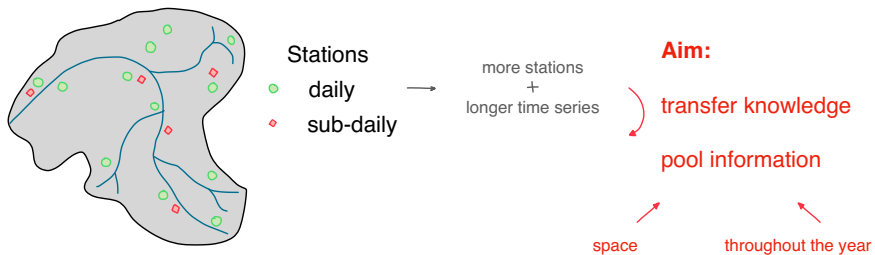
IDF with covariates



Spatial covariates

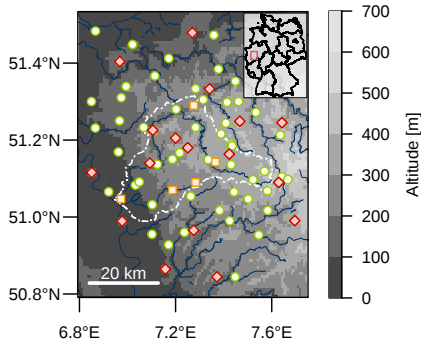




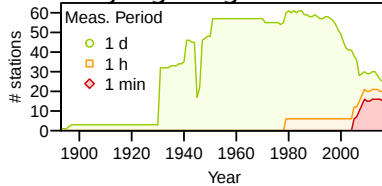


Precipitation Data

- ▶ Case study
Wupper-Catchment:
 - ▶ 92 gauges in 75 locations
 - ▶ provided by DWD and Wupperverband



- ▶ different measuring periods
- ▶ varying length of time series

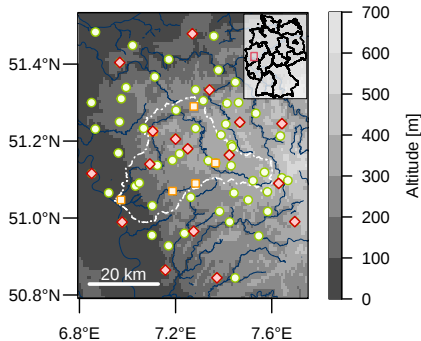


Precipitation Data

► Case study

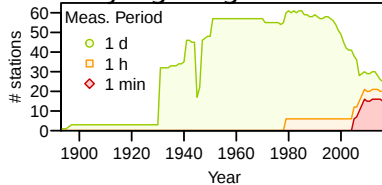
Wupper-Catchment:

- 92 gauges in 75 locations
- provided by DWD and Wupperverband



► different measuring periods

► varying length of time series



Aim:

- model all stations simultaneously
- IDF curves for every location

Research Questions:

- Can we improve IDF estimation?
- Also at ungauged sites?

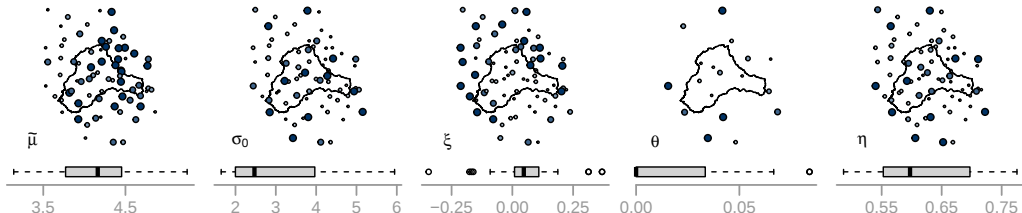
Spatial Covariates

► Model annual maxima for durations

1, 4, 8, 16, 32 minutes,
1, 2, 3, 8, 16 hours and
1, 2, 3, 4, 5 days

using the d-GEV

$$G(z, d; \tilde{\mu}, \sigma_0, \xi, \theta, \eta)$$



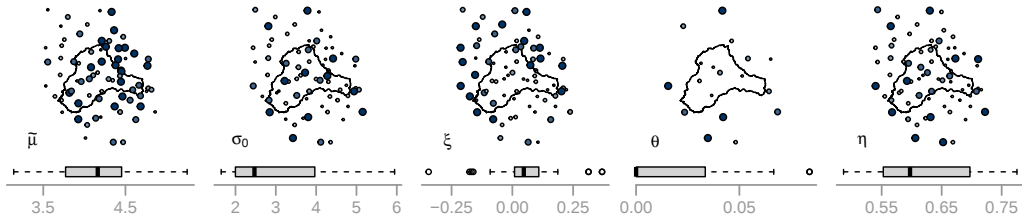
Spatial Covariates

► Model annual maxima for durations

1, 4, 8, 16, 32 minutes,
1, 2, 3, 8, 16 hours and
1, 2, 3, 4, 5 days

at all **locations** using the d-GEV
with **spatial covariates**

$$G(z, d; \tilde{\mu}(\vec{r}), \sigma_0(\vec{r}), \xi(\vec{r}), \theta(\vec{r}), \eta(\vec{r}))$$



Spatial Covariates

- Model annual maxima for durations

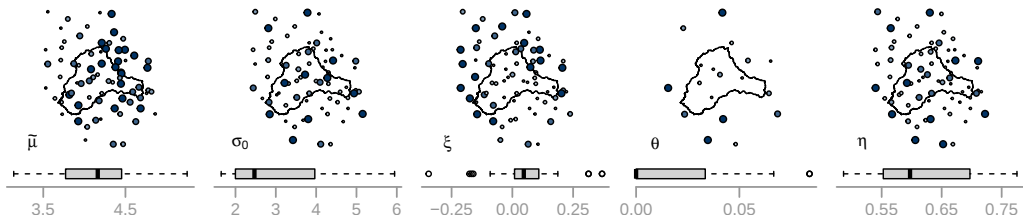
1, 4, 8, 16, 32 minutes,
1, 2, 3, 8, 16 hours and
1, 2, 3, 4, 5 days

at all **locations** using the d-GEV
with **spatial covariates**

$$G(z, d; \tilde{\mu}(\vec{r}), \sigma_0(\vec{r}), \xi(\vec{r}), \theta(\vec{r}), \eta(\vec{r}))$$

- Orthogonal polynomials of lon and lat¹ for all $\phi \in \{\tilde{\mu}, \sigma_0, \xi, \theta, \eta\}$

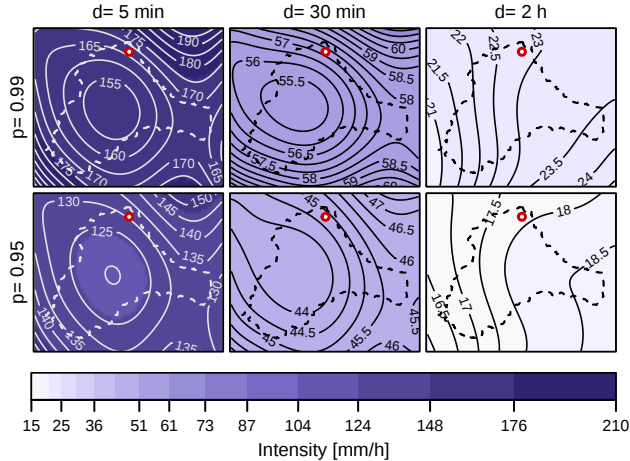
$$\phi(\vec{r}) = \phi_0 + \sum_{j=1}^J \beta_j^\phi P_j(\text{lon}) + \sum_{k=1}^K \gamma_k^\phi P_k(\text{lat}) + \sum_{j=1}^J \sum_{k=1}^K \delta_{j,k}^\phi P_j(\text{lon}) P_k(\text{lat})$$



¹ e.g. Fischer et al. *Spat. Stat.* 2019, 34:100275

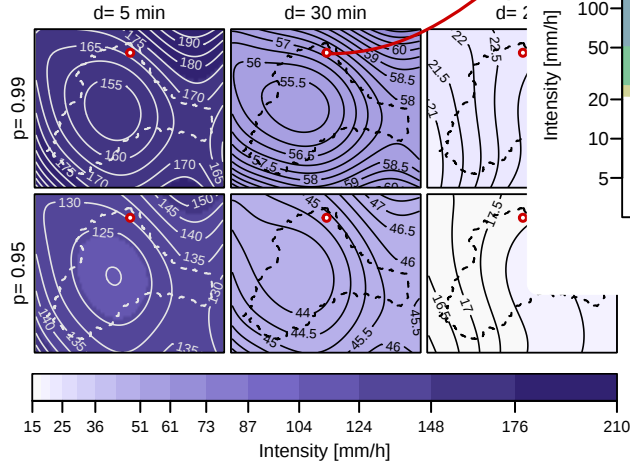
Results

Return levels for various durations and non-exceedance probabilities

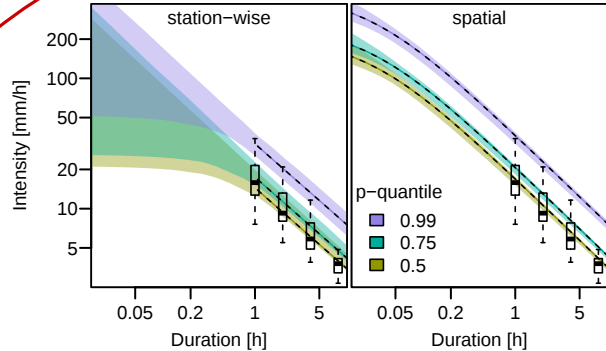


Results

Return levels for various durations and non-exceedance probability

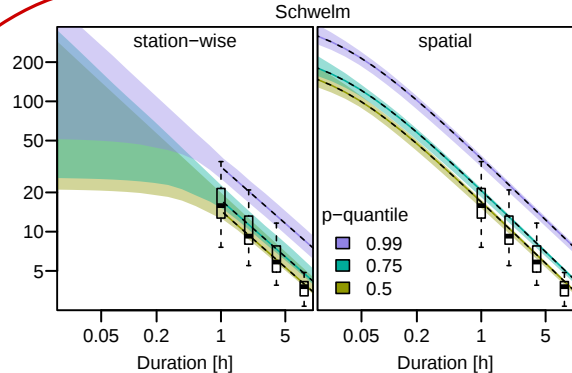
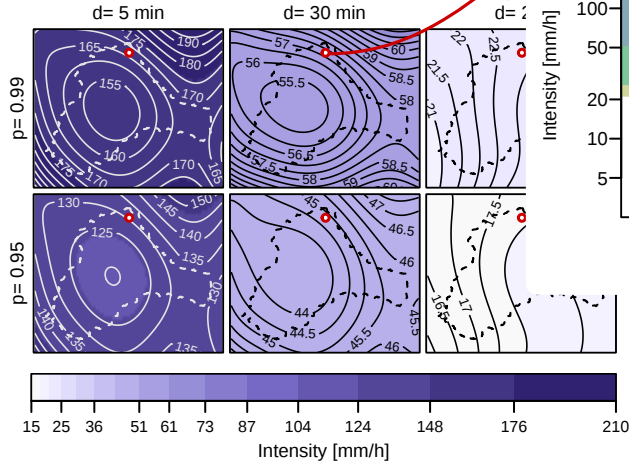


Schwelm

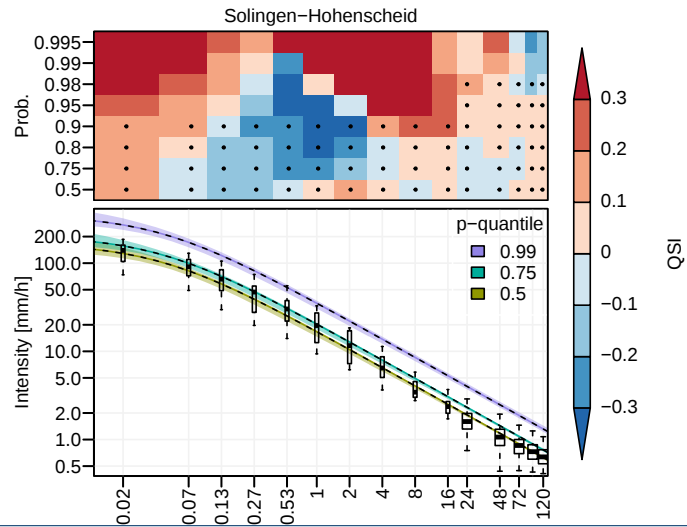


Results

Return levels for various durations and non-exceedance probabilities

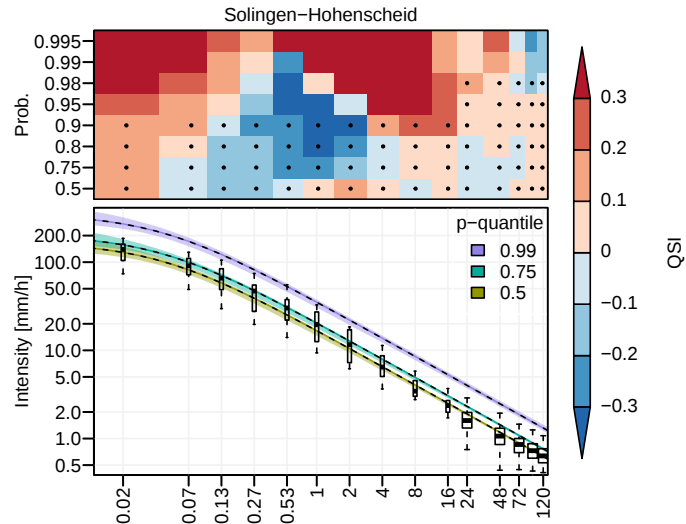


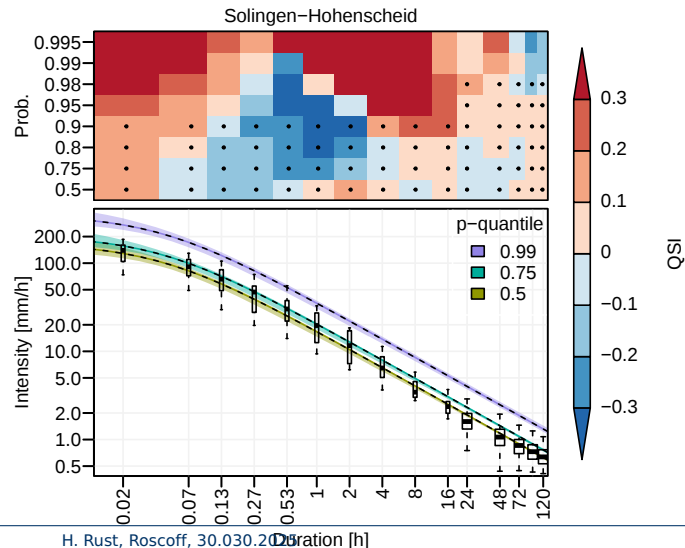
- ▶ reduced uncert.
- ▶ estimates ungauged durations/ locations



Methods

- ▶ Quantile Score: compare modeled quantiles to observations
- ▶ Quantile Skill Index: compare performance of **spatial d-GEV** model with **reference (separate GEV)**





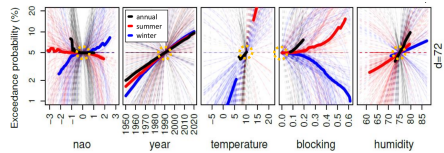
Methods

- ▶ Quantile Score: compare modeled quantiles to observations
- ▶ Quantile Skill Index: compare performance of **spatial d-GEV** model with **reference (separate GEV)**

Results

- ▶ improvement for rare events
- ▶ Improvement for short series
- ▶ $d \geq 24$ h $\rightarrow$ more data, pooling less important
- ▶ not all durations show improvement $\rightarrow$ lack of flexibility

Estimating IDF curves consistently over durations with spatial covariates.
Jana Ulrich, Oscar Jurado, Madlen Peter, Marc Scheibel and Henning Rust
Water, 12(11), 3119
<https://www.mdpi.com/2073-4441/12/11/3119>



IDF with covariates

IDF parameters depending on large scale variables

→ Distribution parameters: function of large-scale variable

Include large-scale information

- polynomial up to 4th order
- each d-GEV parameter can depend on up to two large-scale variables
- Stepwise BIC model selection
- Cross-validated (2-fold)
- No spatial dependence

$f(x)$: Polynomial regression

Examples for dependencies of d-GEV parameters

(scale offset)	$\sigma_0(x)$	$= f(x_1)$
(rescaled location)	$\tilde{\mu}(x)$	$= f(x_2)$
(shape)	$\xi(x)$	$= f(x_1^2)$
(curvature)	$\theta(x)$	$= (constant)$
(duration exponent)	$\eta(x)$	$= f(x_1^2) + f(x_2^3)$
(multiscaling)	$\eta_2(x)$	$= f(x_3)$
(intensity offset, flattening)	$\tau(x)$	$= f(x_1) + f(x_3)$

Example large-scale variables

x_1 - NAO

x_2 - temperature

...

Large scale variables

Temperature and humidity

- ERA5, 0.25°
- 4°W-15°W, 45°N-55°N (average)

Binary Blocking-Index (BBI)

- Scherrer et al, 2006
- ERA5, area mean over SCA

→ all 1950-2015

North-Atlantic Oscill. (NAO) index

- NOAA, PCA-based

→ all averaged over month/year

(Fauer, Rust, 2023)

Large scale variables

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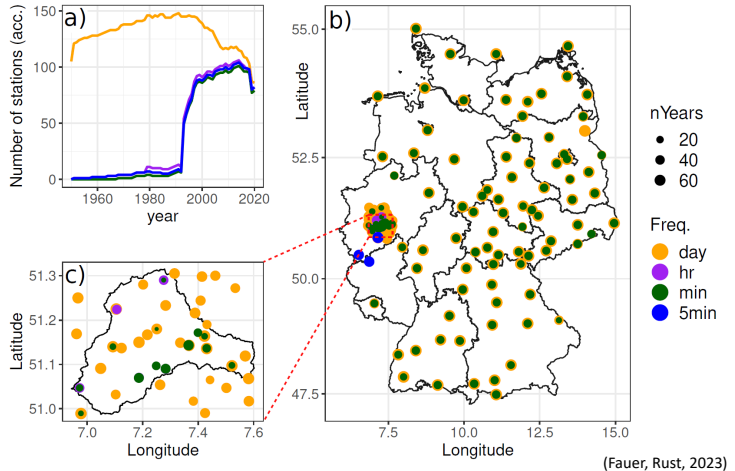
North-Atlantic Oscill. (NAO) index

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Precipitation

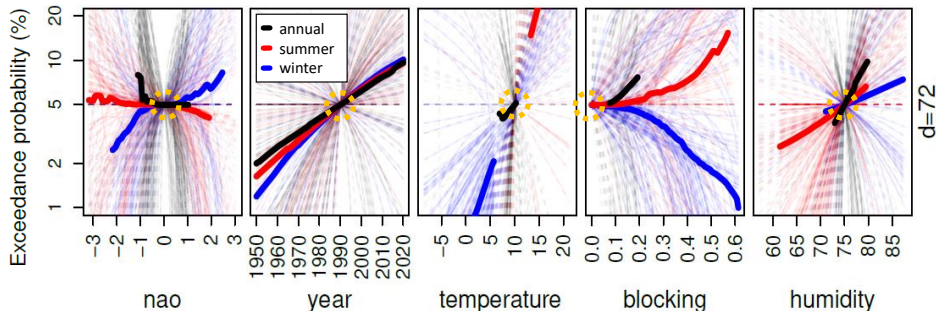
- DWD, Wupperverband
- station-based data
- different temporal resolutions



Results for observed covariate dependence

- define a reference event 
- simulate changes of probability $\updownarrow$ in changing large scale conditions $\leftrightarrow$
- other parameters are fixed

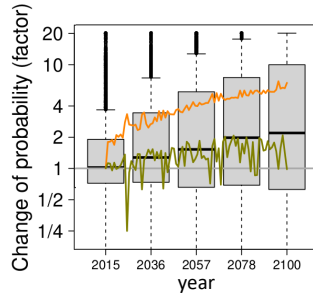
-  Reference event at
- Probability $p = 5\%$
 - NAO $n = 0$
 - Year $y = 1990$
 - Temperature $T = 10^\circ\text{C}$
 - Blocking $b = 0$
 - Humidity $h = 75\%$



(Fauer, Rust, 2023)

Results for projections

- Simulate changes of probability $\updownarrow$ in changing large-scale conditions $\leftrightarrow$
- Projections from MPI-ESM for temperature , humidity, year



Distribution of 197 stations

Summary and Reference

summary

- ▶ IDF depends on atmospheric quantities
- ▶ study processes
- ▶ climate change projections for IDF, 'year' as proxy for climate change

Reference

Non-stationary large-scale statistics of precipitation extremes in central Europe
F. Fauer and H. Rust
Stoch Environ Res Risk Assess 37, 4417–4429 (2023)
<https://doi.org/10.1007/s00477-023-02515-z>