Introduction

- **Title :** Reconstruction of the Evolution Glacier Fronts Trajectories by Stochastic Simulations
- **Project :** PEPR IRIMA (SNA), PC IRIMONT
- **Start :** January 2025
- **Supervisors :** Philippe Naveau (LSCE), Nicolas Eckert (IGE),Mike Pereira (Centre de Géosciences, Mines PSL), Vincent Jomelli (CEREGE)
- **Place :** LSCE (Laboratoire des Sciences du Climat et de l'Environnement) ESTIMR (Extrèmes : STatistiques, Impacts et Régionalisation) team

2. Context

Context - Motivation

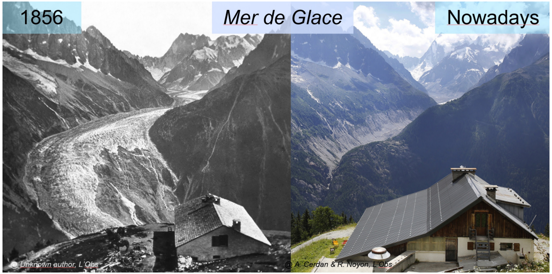


Figure: Mer de Glace in 1856 and now

Context - Definitions

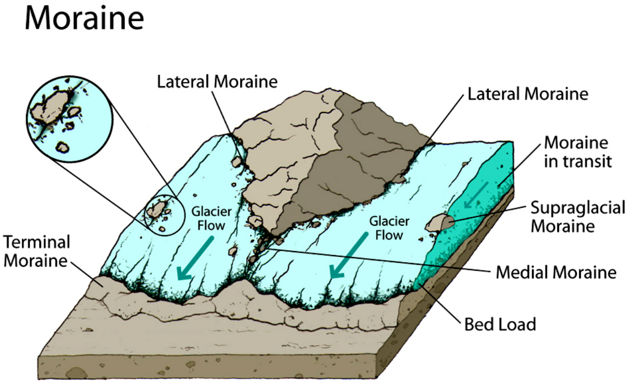


Figure: Moraine formation, source : nationalgeographic

Context - Visual Representation

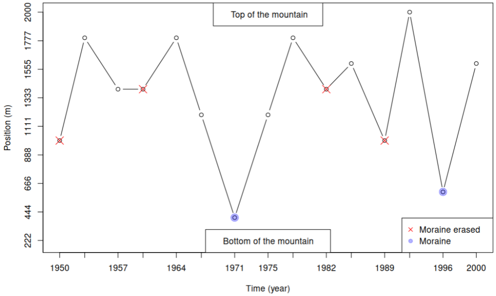


Figure: Front variation evolution

Context - Visual Representation

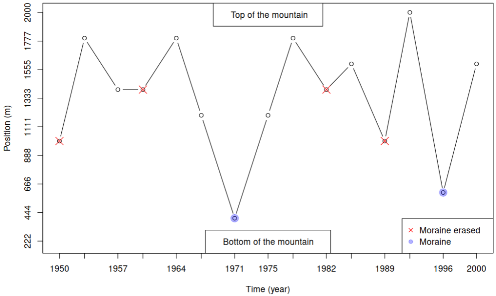


Figure: Front variation evolution

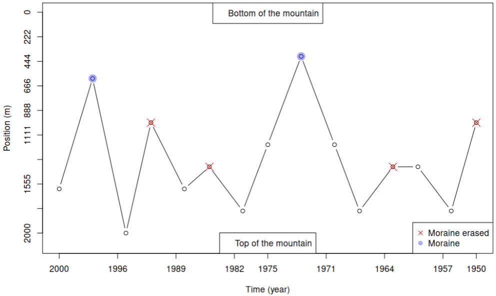


Figure: Front variation evolution reversed

Context - Visual Representation

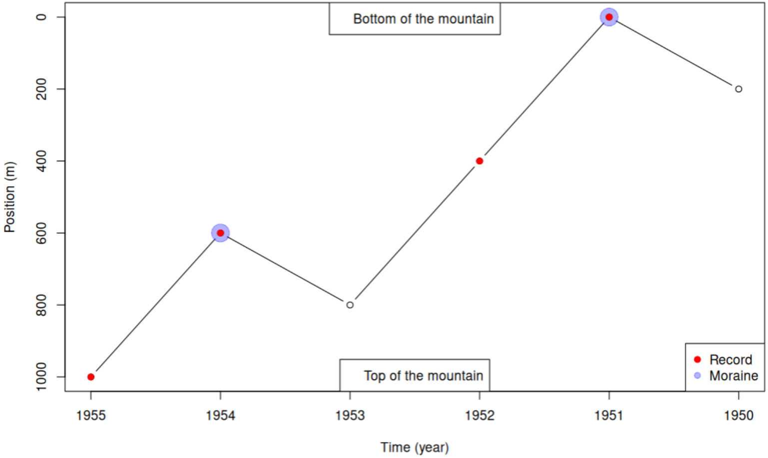


Figure: Special Case

Context - Definitions

Definition : Moraine

The **moraines** correspond to the records of the serie of the front relative positions local maxima when the time axis is reversed.

Context - Bossons Glacier

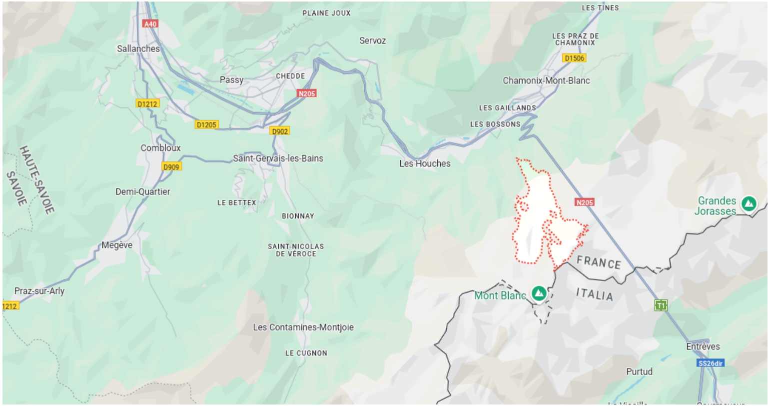


Figure: Map of the Bossons Glacier

Context - Moraines Detection

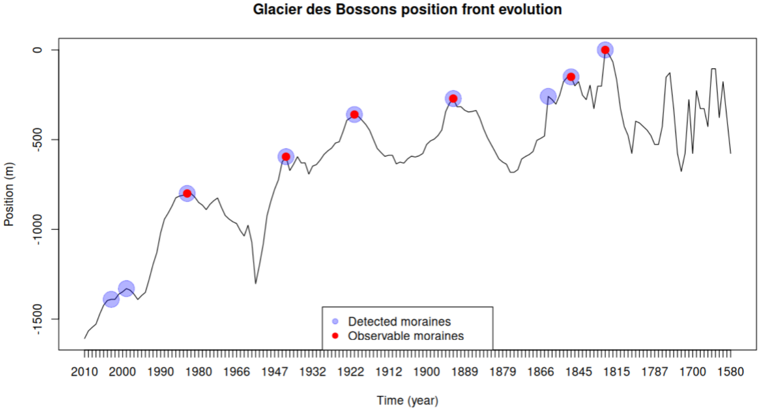


Figure: Comparaison between the observed moraines for the Glacier des Bossons and those algorithmically detected

3. Methods

Method - Objectives

Spatio-temporal evolution reconstruction based on observable moraines

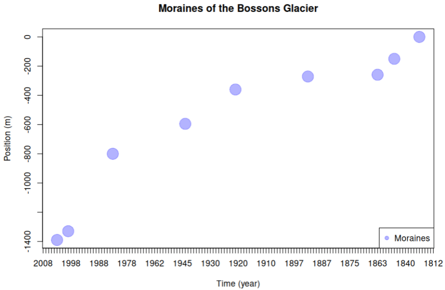


Figure: Observable moraines

Method - Objectives

Spatio-temporal evolution reconstruction based on observable moraines

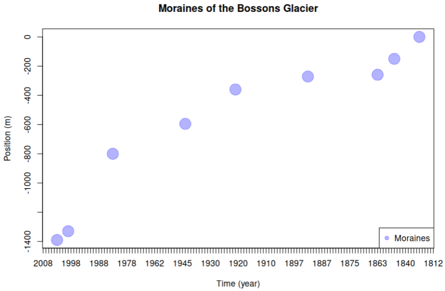


Figure: Observable moraines

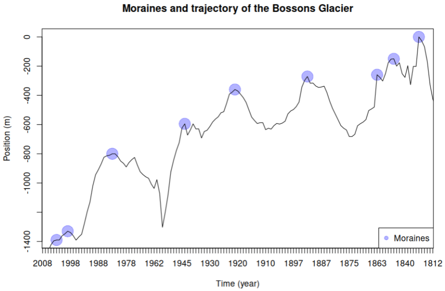


Figure: What we want to build

Method - Challenges

- How to use the **theory of extreme values and records** to construct, infer and validate the model ?
- How to take into account the non-stationarity ?
- **Glaciological point of view** : what can be said about long-term front trajectories independently of climate forcing data?
- What is the specificity of trajectories as a function of **local characteristics** (slope, glacier size, ...) or regional context (Alps, Himalayas, ...)?

Method - Steps

- Construct a **probabilistic model** respecting the physical constraints of the moraines
- Develop some **inference techniques** to evaluate the parameters
- Evaluate and validate it on available complete trajectories data
- Improve it (missing moraines, imprecise dating, spatial context)
- Apply it to existing moraine series
- Interpret it in an interdisciplinary manner

Method - Data

Data used for this project for the construction and validation of the model :

- Recent data (less than 200 years)
- 25 glaciers in France and Switzerland
- Annual front variation (trajectory)

Figure: Bossons Glacier (FR)

Method - Explored Methods

- Conditionned brownian Motion
- Folding
- Gaussian Conditioning

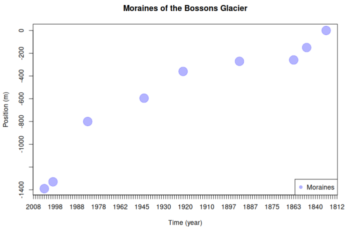


Figure: Observable moraines

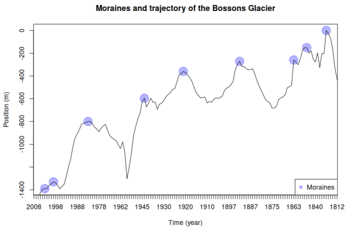


Figure: What we want to build

3. Brownian Motion

Brownian Motion - Definition

Definition : Brownian Motion X

$(X_t)_{t\geq 0}$ is a stochastic process verifying :

- $(X_t)_{t\geq 0}$ is a Gaussian process
- $(X_t)_{t\geq 0}$ is almost surely continuous
- $\forall (s,t) \in \mathbf{R}_+^2, \mathbb{E}[X_t] = 0, \mathbb{E}[X_s X_t] = \min(s,t)$.

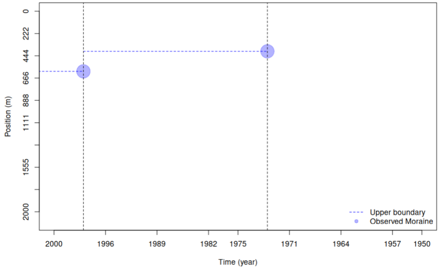
Can be seen as a random walk : Markov property

Brownian Bridge : Brownian motion which is equal to zero at the start and end time points.
Brownian excursion : Nonnegative Brownian Bridge.

Brownian Motion - Applications

Method : Simulation between moraines

- Cutting into pairs of successive moraines
- Simulation of a Brownian trajectory between each pair
- Regrouping trajectories
- (Smoothing)



Brownian Motion - Results

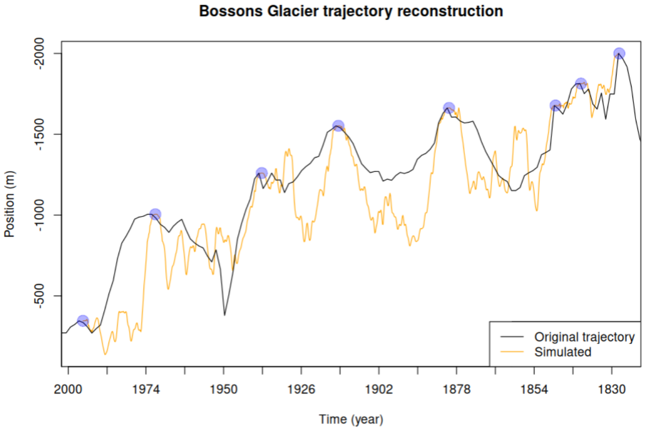


Figure: Simulated trajectory using Brownian Motions

4. Folding

Folding - Principle

Goal : Simulate a random variable X conditioned to $X \in [a, b]$
Pros : Generic approach which respects to the conditional distribution, not rejection-based
Method : Use the (unconditioned) distribution function of X to define a new variable $X^{(F)}$ with values in $[a, b]$.
Source : Improving extreme quantile estimation via a folding procedure, You et al. 2010

Folding - Definition

F the distribution function of X

$$X^{(F)}(a,b) = \begin{cases} F^{-1}(\frac{F(X)}{F(a)}(F(b)-F(a))+F(a) & \text{if } X < a \\ F^{-1}(\frac{\bar{F}(X)}{\bar{F}(b)}(F(a)-F(b))+F(b) & \text{if } X > b \\ X & \text{otherwise.} \end{cases} \quad (1)$$

Proposition

For $a < x < b$:

$$P(X^{(F)}(a,b) > x) = P(X > x | a < X < b) \quad (2)$$

Folding - Definition

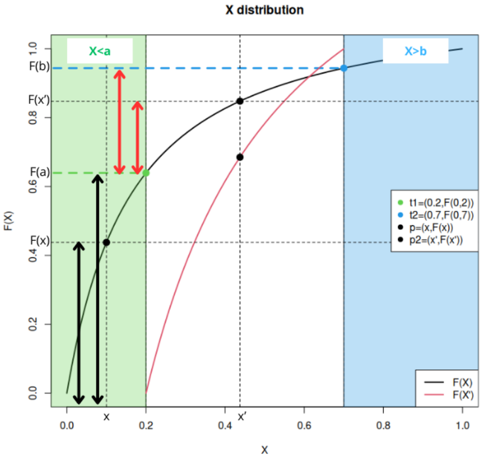


Figure: Folding method

Folding - Applications

Method : Folding between moraines

- Simulate a trajectory X
- Fold X between each moraine with, for $i \in [\![1, k]\!]$:
  - a_i lower boundary : first we take 0
  - b_i upper boundary : the moraines
  - $X[t_{i-1}, t_i]$ observations to fold
- Assemble each folded part

Folding - Results

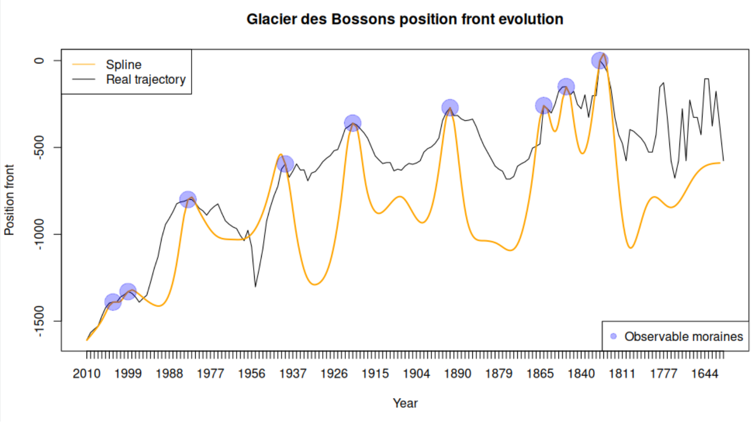


Figure: Results on the Glacier des Bossons trajectory

5. Gaussian Conditional Distribution

Gaussian Conditioning - reminders

What was done :

- Simulation of X_1 representing the glacier positions
- Detection of the X_1 moraines (records of the local maxima)
- Simulation of another random variable X_2 following the same law as X_1
- Folding of X_2 in order to respect the X_1 moraines

Problem encountered : We use the parameters of the law of X_1 to simulate X_2 but in practice we don't know them

Gaussian Conditioning - Theory

Let $X = \begin{pmatrix} X_t \\ X_d \end{pmatrix} \sim \mathcal{N}(\begin{pmatrix} \mu_t \\ \mu_d \end{pmatrix}, \begin{pmatrix} \Sigma_{tt} & \Sigma_{td} \\ \Sigma_{dt} & \Sigma_{dd} \end{pmatrix})$
With the conditional distribution:

$$(X_t | X_d = x_d) \sim \mathcal{N}(m_{t|d}, S_{t|d}) \tag{3}$$

With :

$$m_{t|d} = \mu_t + \Sigma_{td}\Sigma_{dd}^{-1}(x_d - \mu_d) \tag{4}$$

And :

$$S_{t|d} = \Sigma_{tt} - \Sigma_{td}\Sigma_{dd}^{-1}\Sigma_{dt} \tag{5}$$

Gaussian Conditioning - Solution

Principle: Simulate a random variable by imposing certain values on a subset of its observations
Application : Simulate a random variable by forcing it to pass through the observed moraines, using the **Matern covariance** (shape parameter to control the degree to which the underlying Gaussian field is differentiable, and the degree to which trajectories are smooth)

Gaussian Conditioning - Applications

Trajectory simulation algorithm :

- Given a set of moraines $X_m = x_m$
- Use conditional simulation to create X point by point :
  For $i \in [\![1, n]\!]$:
  - Simulate a single point x_i conditionally on all previous points $x_{p,i} = \{x_j, j < i\}$ and the moraines $x_{d,i} = x_m \cup x_{p,i}$
  - Fold this point into x_i^f to make sure it is inferior to the next moraine value
  - Modify the subset used to simuate the next point $x_{p,i+1} = x_{p,i} \cup x_i^f$

Gaussian Conditioning - First results

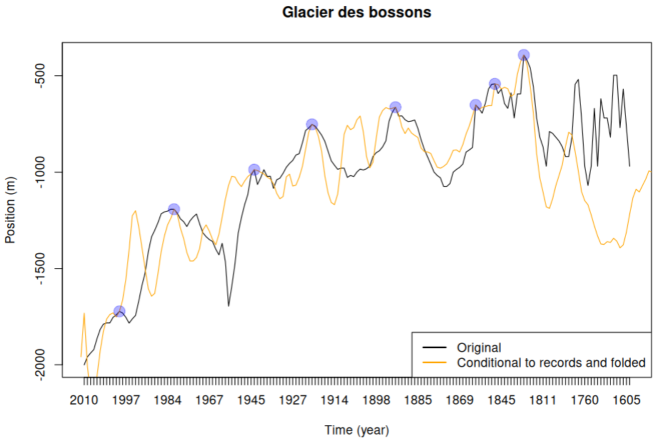


Figure: Gaussian Conditioning and Folding first result on the Bossons Glacier

6. Conclusion and discussion

Conclusion

Method	Pros	Cons
Brownian Motion	No post modification needed	
Folding	Avoid acceptance rejection	Information about trajectories needed
Gaussian Conditioning	Gaussian properties We create X point by point Only moraines values needed	Time consuming

Discussion

- Which physical constraints should be considered ? (Volume, slope, climate data ?)
- How to compare simualted trajectories ? (score)
- What about not forcing moraines but simulate them ?

Thank you for your attention !

Brownian Motion - Definition

X a brownian bridge
Construction of a Brownian excursion e_t:
$\tau = Argmin(X_t)_{t\in[0,1]}$
Verwaat transformation :

$$e_t = X_{\tau+t} - X_\tau, t \in [0,1] \quad (6)$$

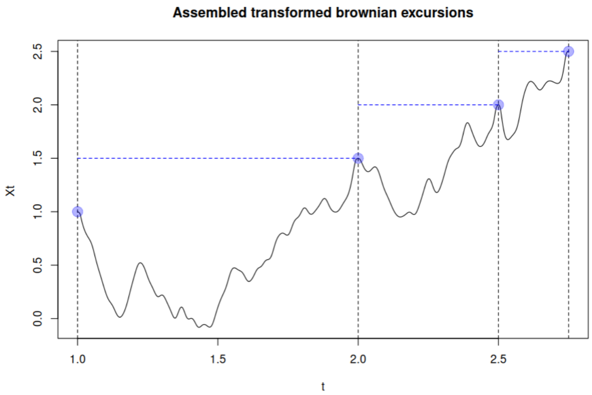


Figure: Example of 3 browian excursions